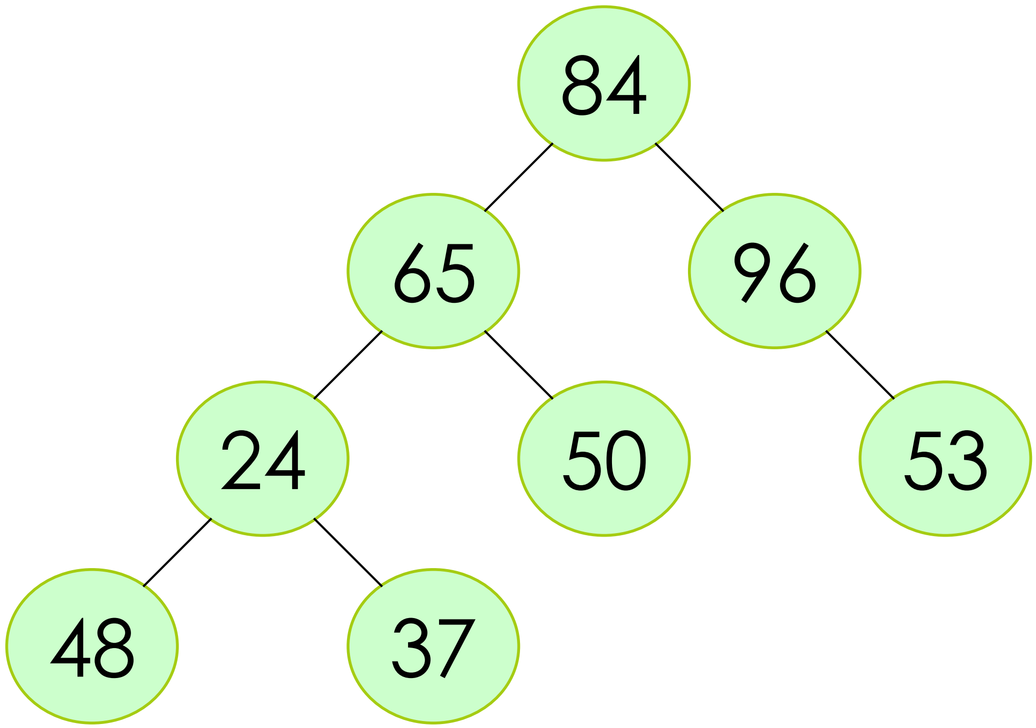
Data Structures:
Trees and Graphs

Trees

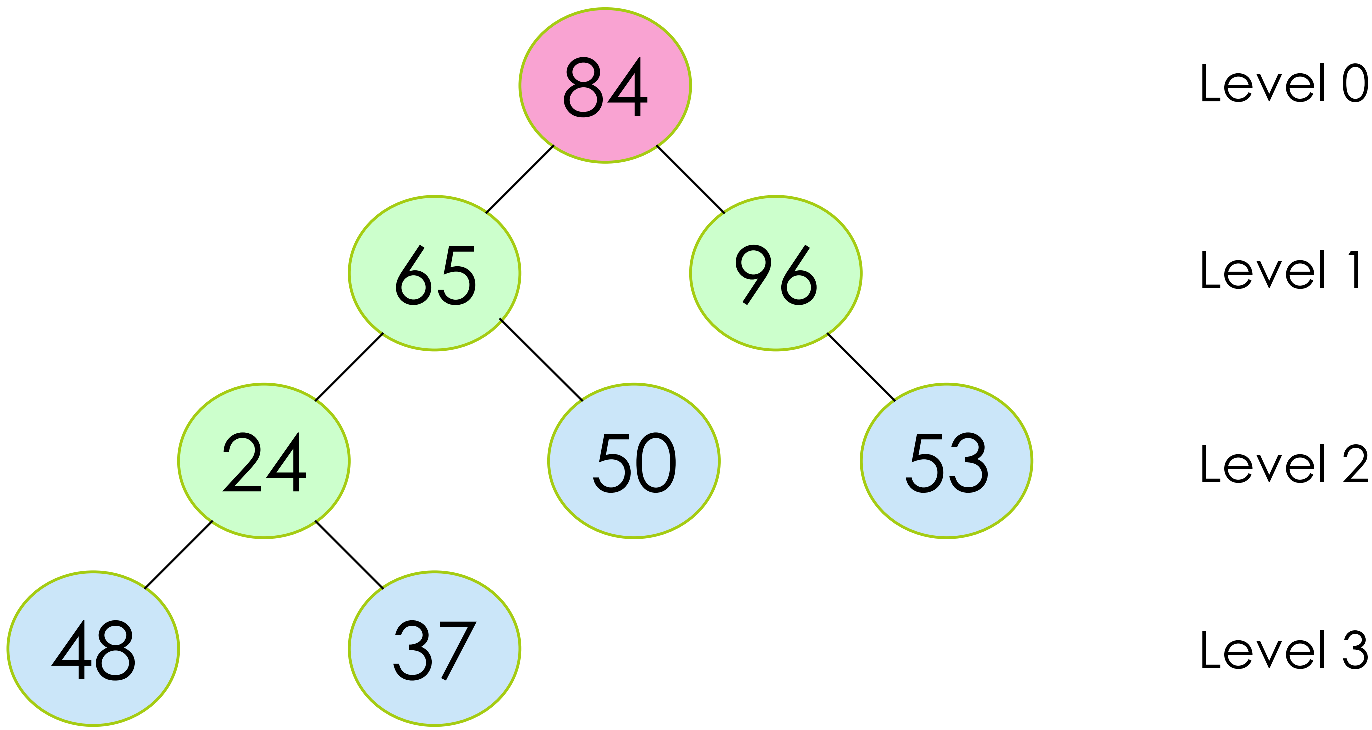
- A **tree** is a hierarchical data structure composed of **nodes**.
  - **Root:** the top-most node (unlike real trees, trees in computer science grow downward!). Every (non-empty) tree has one.
  - **Parent:** the node connected directly above the current one. Every node (except for the root) has one.
  - **Child:** a node connected below the current one. Each node can have 0 or more.
  - **Leaf:** a node that has no children.
  - **Depth/Level:** the length of the path (edges) from the root to a node (depth/level of the root is 0).
  - **Tree Height**: the maximum depth from of any node in the tree.
- A tree commonly used in computing is a **binary tree**.
  - A binary tree consists of nodes that have at most 2 children.
  - Commonly used in: data compression, file storage, game trees

Binary Tree Example



Which node is the root?
Which nodes are the leaves?
Which nodes are internal nodes?
What is the height of this tree?

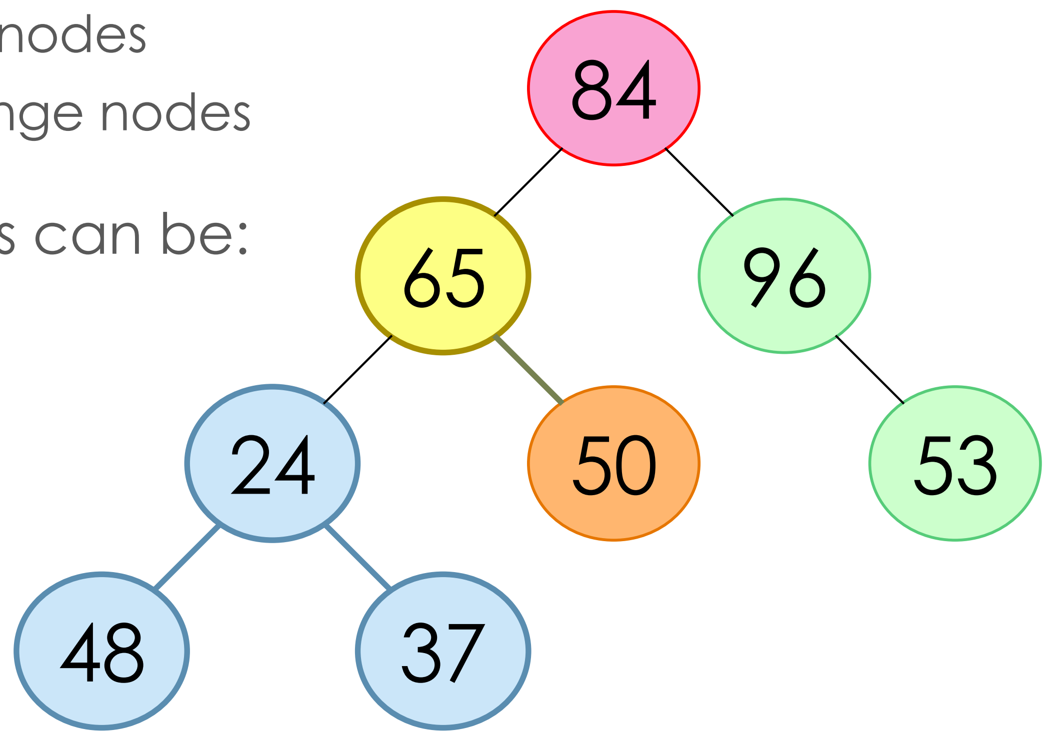
Binary Tree Example



The root contains the data value **84**.
There are 4 leaves in this binary tree: nodes containing **48, 37, 50, 53**.
There are 4 internal nodes in this binary tree: containing **84, 65, 96, 24**
This binary tree has **height 3** – considering **root is at level 0**,
the **length of the longest path from root to a leaf** is **3**

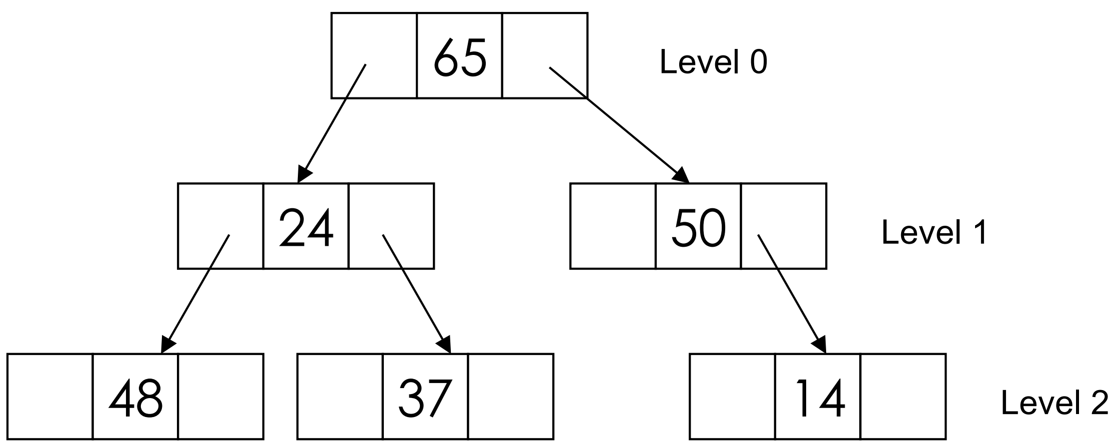
Binary Trees: A recursive structure!

- The yellow node with the key 65 can be viewed as the **root** of the left subtree, which in turn has
  - a left subtree of blue nodes
  - a right subtree of orange nodes
- In general, Binary Trees can be:
  - Empty
  - A root node with
    - a left binary tree
    - a right binary tree



Binary Trees: Implementation

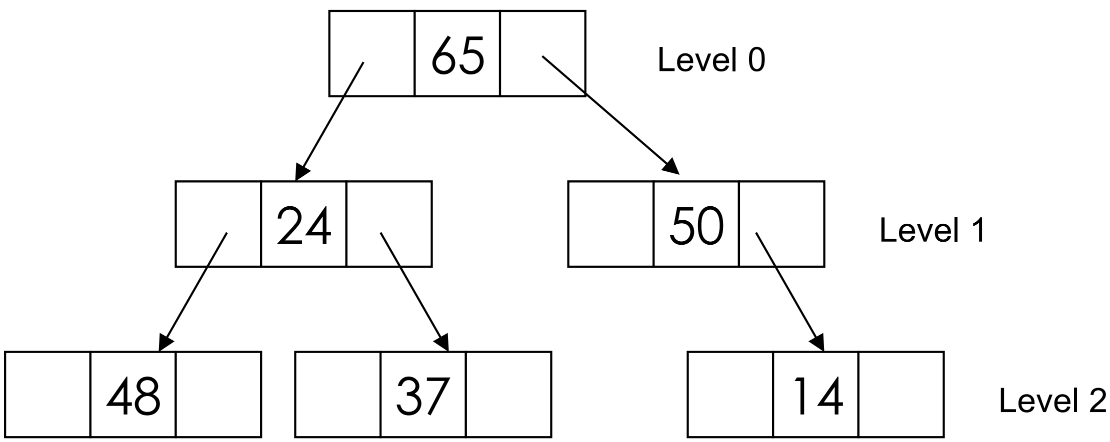
- A common implementation of binary trees uses nodes
  - Each node has a "left" node and a "right" node.
- How to represent these nodes and pointers? With a Class (like a Struct…)



Implement Using a List

- We could also use a list to implement binary trees. For example:

0	1	2	3	4	5	6	7
-	65	24	50	48	37	-	14



Binary Search Tree (BST)

- A **binary search tree (BST)** is a binary tree with no duplicate nodes that imposes an *ordering* on its nodes.
- BST ordering invariant: At **any** node *n* with value *k*,
  - all values of nodes in the left subtree of *n* are strictly **less** than *k*
  - all values of nodes in the right subtree of *n* are strictly **greater** than *k*

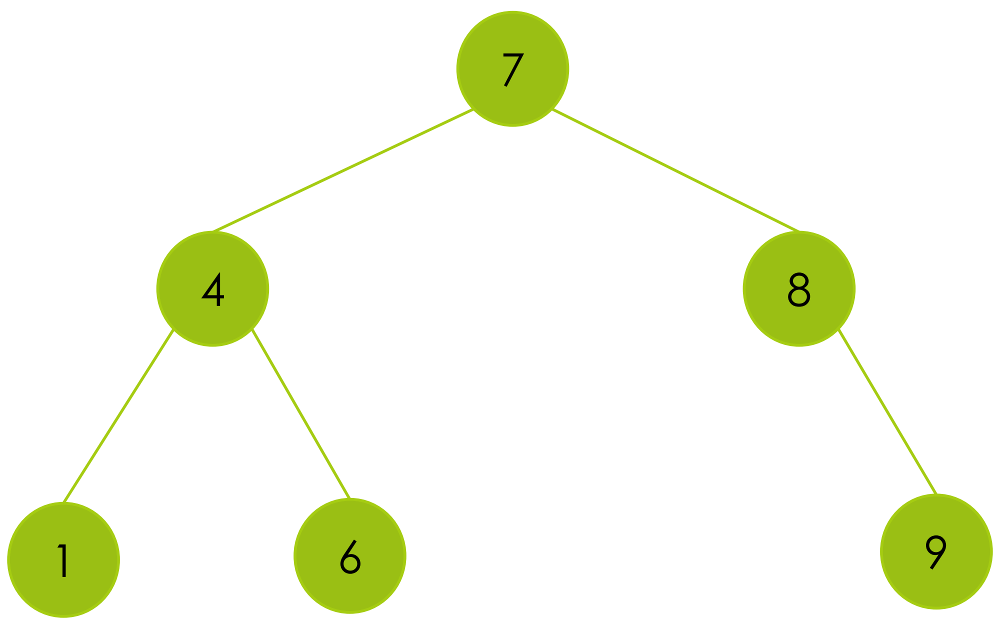
Example: Binary Search Tree

BST ordering invariant: At **any** node with value *k*,

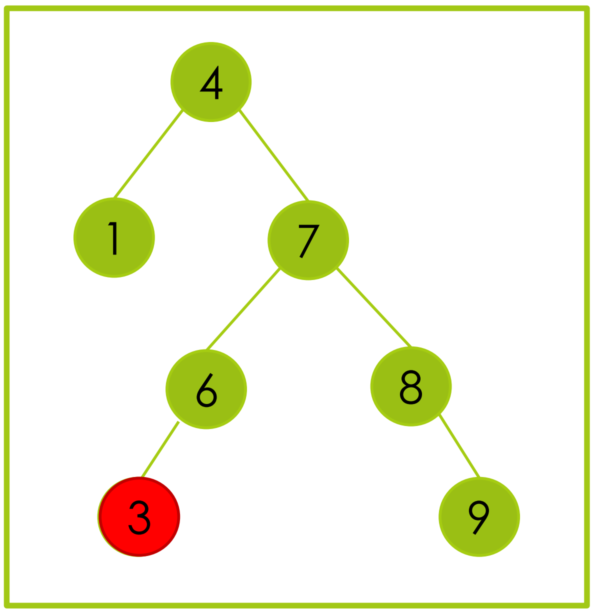
all values of elements in the left subtree are strictly less than *k* and

all values of elements in the right subtree are strictly greater than *k*

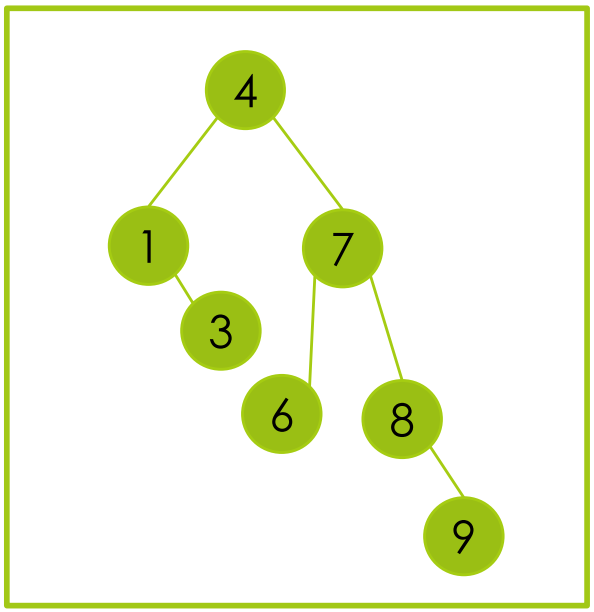
(assuming that there are no duplicates in the tree)



Example: Is this a BST?



no



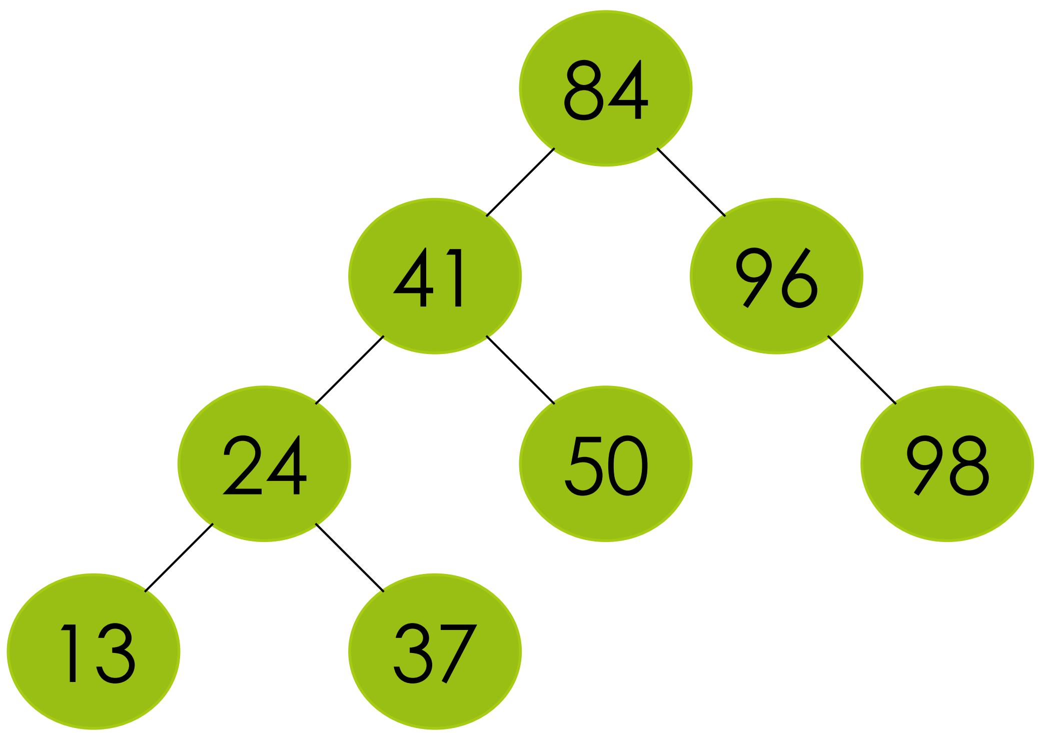
yes

Insertion in a BST

- For each data value that you wish to insert into the binary search tree:
  - If you reach an empty tree (must test this first, why?), create a new leaf node with your value at that location
  - Recursively search the BST for your value until you either find it or reach an empty tree
  - If you find the value, throw an exception since duplicates are not allowed

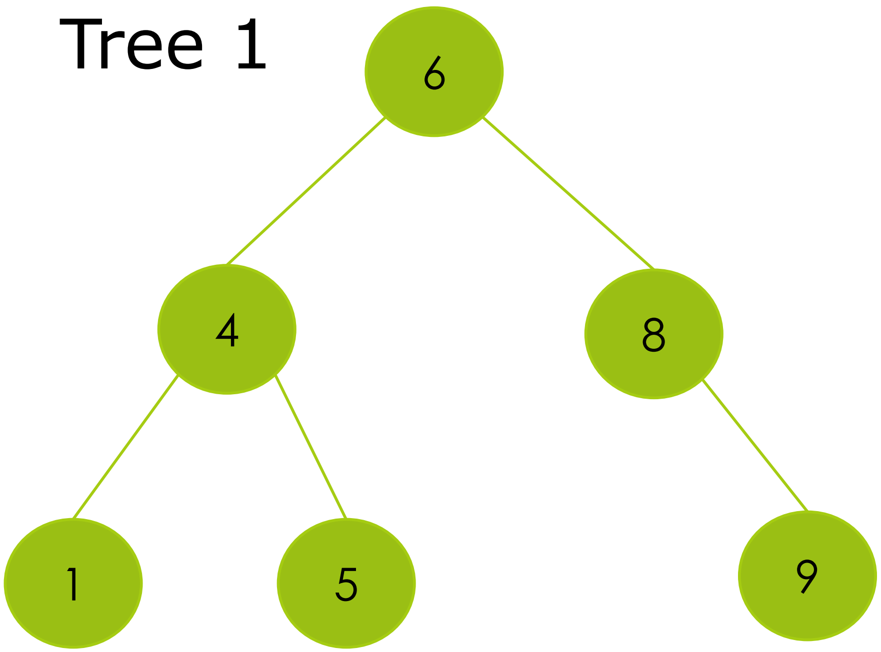
Insertion Example

- Insert: 84, 41, 96, 24, 37, 50, 13, 98



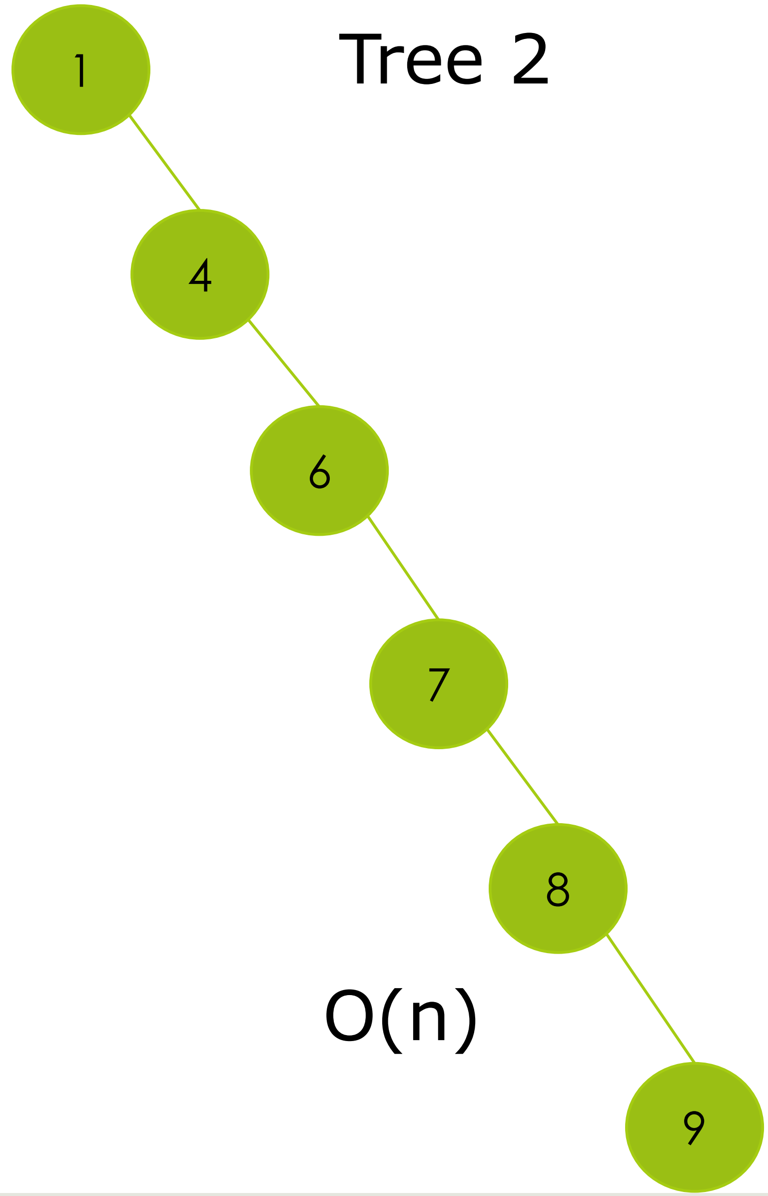
Binary Search Tree Complexity

Tree 1



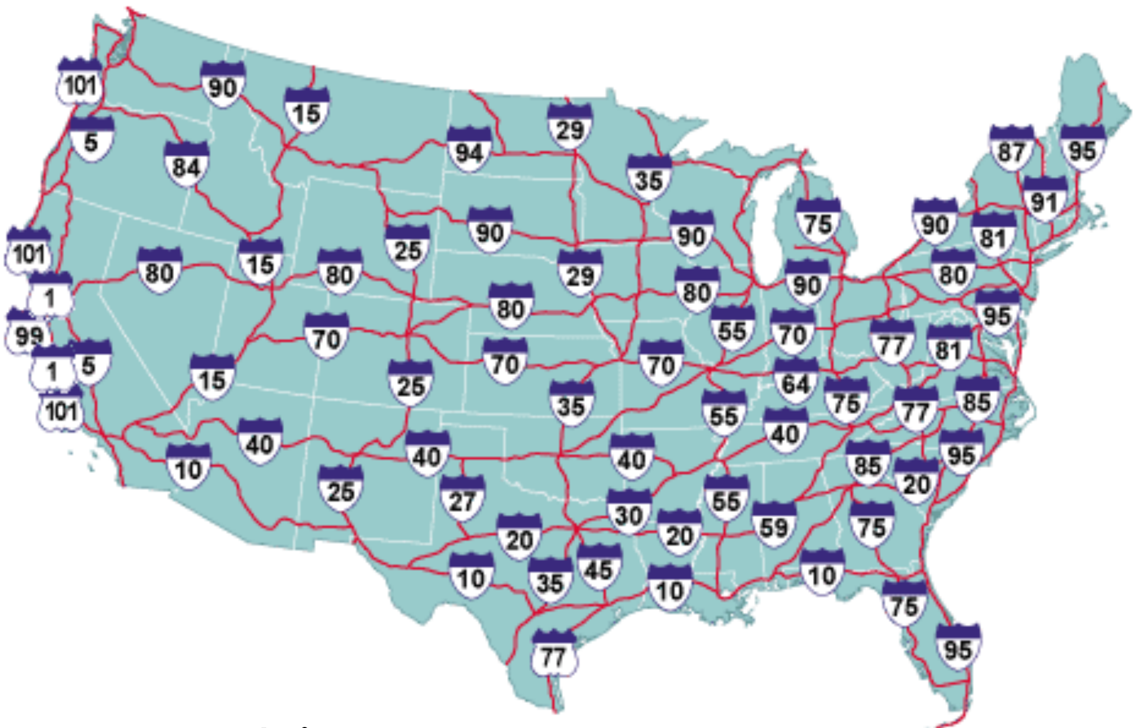
O(log n)

Tree 2

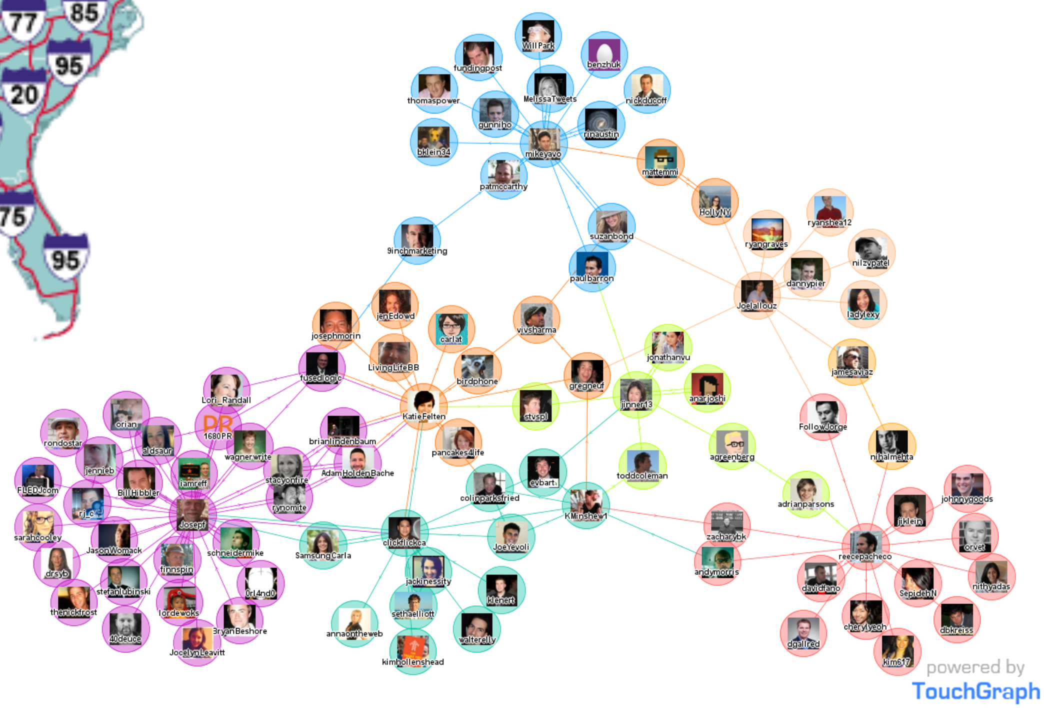


O(n)

Relationship Data

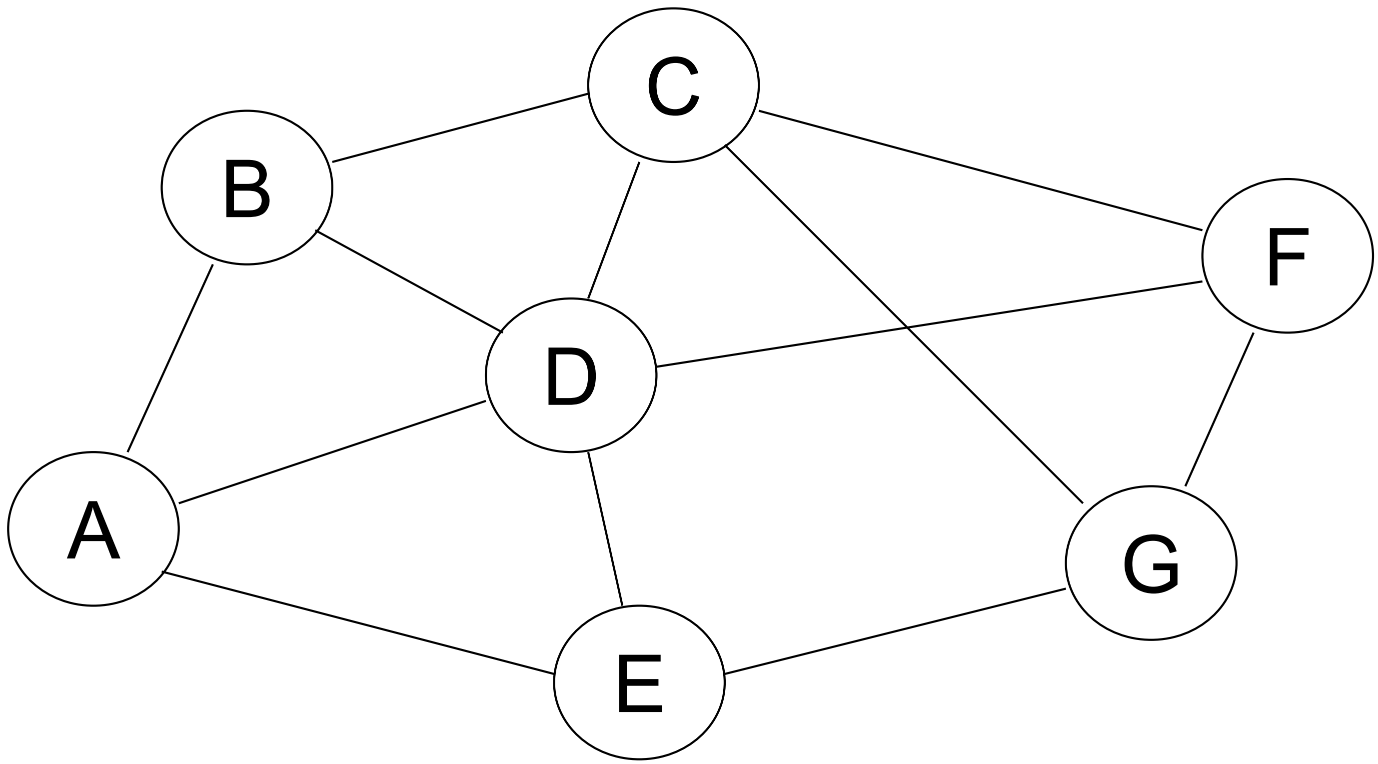


From this…



Relationship Data

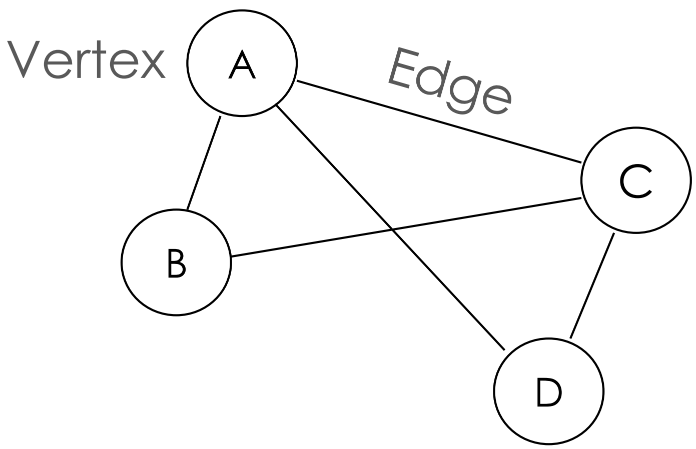
To this…



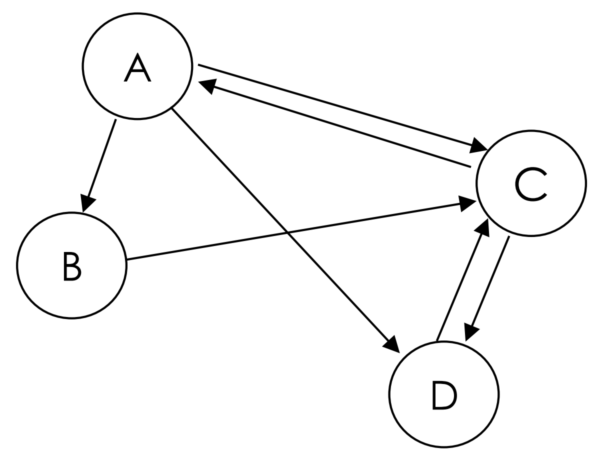
Graphs

- A graph is a data structure that contains of a set of **vertices** and a set of **edges** which connect pairs of the vertices.
  - A vertex (or node) can be connected to any number of other vertices using edges.
  - An edge may be bidirectional or directed (one-way).
  - An edge may have a **weight** on it that indicates a cost for traveling over that edge in the graph.
- Unlike trees, graphs can contain *cycles*
  - In fact, a tree is an *acyclic graph*
- Applications: computer networks, transportation systems, social networks

Example Graphs



Undirected

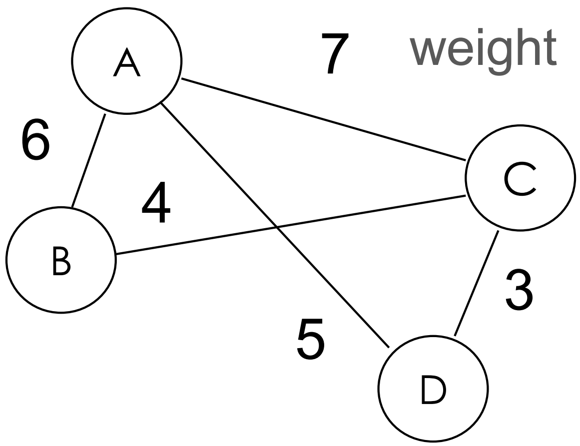


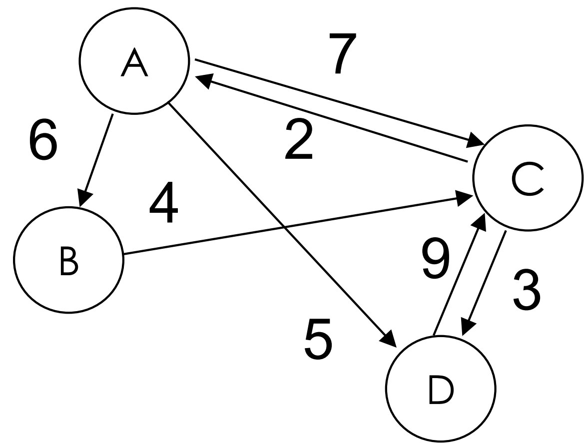
Directed

Graph Implementation

- We usually represent graphs using a *table* (2d list) where each column and row is associated with a specific vertex. This is called an **adjacency matrix**.
- A separate list of vertices shows which vertex name (city, person, etc.) is associated with each index
- The values of the 2d list are the weights of the edges between the row vertices and column vertices
  - If there is not an edge between the two vertices, we use infinity, or None, to represent that

Graph Representation

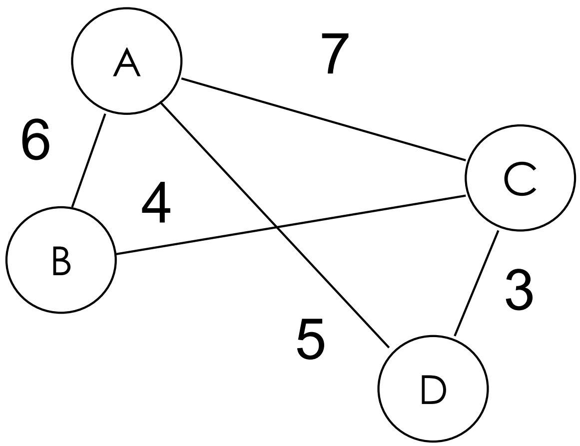




from \ to	A	B	C	D
A	0	6	7	5
B	6	0	4	∞
C	7	4	0	3
D	5	∞	3	0

from \ to	A	B	C	D
A	0	6	7	5
B	∞	0	4	∞
C	2	∞	0	3
D	∞	∞	9	0

Graphs in Python



from \ to	A	B	C	D
A	0	6	7	5
B	6	0	4	∞
C	7	4	0	3
D	5	∞	3	0

```
vertices = ['A', 'B', 'C', 'D']
graph =
[ [ 0, 6, 7, 5 ],
  [ 6, 0, 4, None],
  [ 7, 4, 0, 3],
  [ 5, None, 3, 0] ]
```

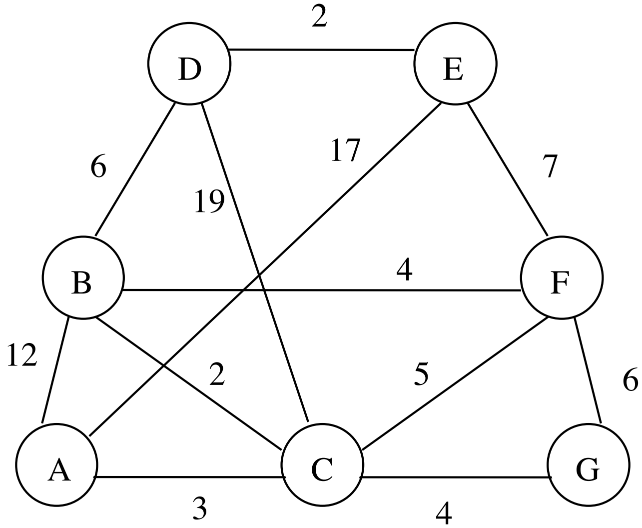
Graph Algorithms

- Lots! Here are some examples.
- There are algorithms to search graphs efficiently for a value
  - Breadth-first search and Depth-first search
- There are algorithms to compute the shortest path between a start vertex and all the others
  - Dijkstra's algorithm
- There are algorithms for operations research, which can be used to solve *network flow* problems
  - For example, how to efficiently distribute water through a system of pipes

Shortest Path (Dijkstra's algorithm)

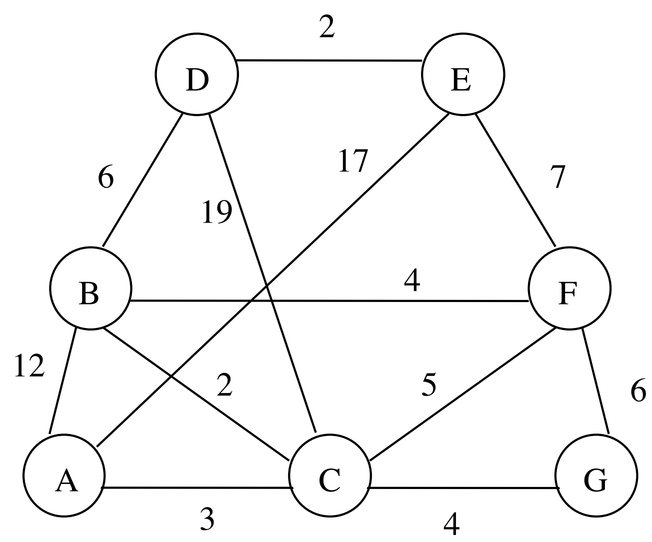
- Assign every node an initial distance (0 to the source, ∞ for all others); mark all nodes as unvisited
- While there are unvisited nodes:
  - select unvisited node with smallest distance (current)
  - consider all unvisited neighbors of current node:
    - compute distance to each neighbor from current node
    - if less than current distance, replace with new distance
  - mark current node as visited (and never evaluate again)

Dijkstra example



	A	B	C	D	E	F	G
	0	∞	∞	∞	∞	∞	∞
A->							

Dijkstra example



	A	B	C	D	E	F	G
	0	∞	∞	∞	∞	∞	∞
A->		12	3	∞	17	∞	∞
C->		5		22	17	8	7
B->				11	17	8	7
G->				11	17	8	
F->				11	15		
D->					13		
E	0	5	3	11	13	8	7

What does this assume?